Code No. 2852/ CDE

FACULTY OF SCIENCE

M. Sc.(Previous) (CDE) Examination, July 2018

Subject : Mathematics

Paper – II : Real Analysis

Time : 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

1 If X is a metric space and E Ċ X then prove that $\bar{E}$ is closed.

2 If $\{I_n\}$ is a sequence of intervals in R such that $I_n \supset I_{n+1,}$ n = 1, 2,……, then prove that $\bigcap_{n=1}^{\infty} I_n$ is not empty.

3 Show that rearrangement of a convergent series need not be convergent through an example.

4 If f is a continuous mapping of a compact metric space X into R^k then prove that f is bounded.

5 If f $\in$ R(α) on [a, b] then prove that |f| $\in$ R(α) and $\left|\int_a^b f\, d\sim\right| \le \int_a^b |f|\, d\sim$.

6 If P* is a refinement of P then prove that
U(P*, f, α) $\le$ U(P, f, α).

7 If $\{f_n\}$ I s a sequence of functions defined on E and if $|f_n(x)| \le M_n$, x $\in$ E, n = 1, 2, …., then prove that Σf_n converges uniformly on E if ΣM_n converges.

8 Show that a convergent series of continuous functions can have a discontinuous sum.

PART – B (5 x 12 = 60 Marks)
(Essay Answer Type)

9 (a) Prove that every k-cell is compact.

OR

(b) Explain the construction of cantors set and prove that cantor set is perfect.

10 (a) If E is a non compact set in R then show that there exists a continuous function on E
which is not bounded.

OR

(b) If f is continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X then prove that f(E) is connected.

11 (a) If f$\in$R(α) on [a, b], m $\le$ f $\le$ M, ϕ is continuous on [m, M] and h(x)=ϕ(f(x)) on [a, b] then prove that h$\in$R(α) on [a, b].

OR

(b) If γ's continuous on [a, b] then prove that r is rectifiable and also prove that

$\Lambda(\gamma)=\int_a^b |¥'(t)|\,dt$.

12 (a) If K is compact, {f_n} is a sequence of continuous functions on K and {f_n} converge pointwise to a continuous function f on K and also $f_n(x) \geq f_{n+1}(x)$ $\forall$ x $\in$K, n = 1, 2,... then prove that $f_n \rightarrow$ f uniformly on K.

OR

(b) State and prove Weierslrass approximation theorem.

13 (a) If {f_n} is a sequence of differentiable functions on [a, b] and {$f_n(x_o)$} converge for some
point $x_o \in$ (a, b) and also if { f_n'} converge uniformly on [a, b] then prove that {f_n} converge uniformly on [a, b] to f and also $f'(x) = \lim_{n\to\infty} f_n'(x)$ on [a, b].

OR

(b) State and prove contraction principle.

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Code No. 2857 / CDE / O / BL

FACULTY OF SCIENCE

M. Sc.(Previous) (CDE) Examination, July 2018

Subject : Mathematics

Paper – II : Analysis

Time : 3 Hours **Max. Marks: 100**

Note : Answer any five questions. All questions carry equal marks.

1 (a) Define a countable and an uncountable set. Prove that the union of a countable number of countable sets is countable.

(b) Define a compact set. Suppose $K \subset Y \subset X$. Prove that K is compact relative to X if and only if K is compact relative to Y.

2 (a) Prove that every K-cell is compact.

(b) Prove that compact subsets of metric spaces are closed.

3 (a) Prove that in a compact metric space X, every Cauchy's sequence in X converges to some point in X.

(b) State and prove the Reimann's rearrangement theorem.

4 (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

(b) Prove that monotonic functions have no discontinuities of the second kind.

5 (a) Define the upper and lower Riemann Stieltjes integrals of a bounded function f defined on [a, b]. Show that

$$\int_{-a}^{b} f \; d\sim \;\le \int_{a}^{b} f \; d\sim$$

(b) Prove that $f \in R(x)$ on [a, b] if and only if for every $\in > 0$ there exists a partition P of [a, b] such that $U(p, f, \alpha) - L(p,f,\alpha) < \in$

6 (a) Suppose $f \in R(\alpha)$ on [a, b] and c is any constant prove that $C f \in R(\alpha)$ on [a, b] and $\int_a^b C\, f\; d\sim = C\int_a^b f\; d\sim$. In particular if C is a positive constant prove that

$f \in R(\alpha)$ and $\int_a^b f\; d(C\sim) = C\int_a^b f\; d\sim$

(b) Let $f \in \mathbf{R}$ on [a , b]. For $a \le x \le b$ let $F(x) = \int_a^x f\,(t)\,dt$

prove that F is continuous on [a, b]. Also prove that if f is continuous at a point x_o of [a, b] then F is differentiable at x_o and $F'(x_o) = f(x_o)$.

7 (a) Suppose $\lim_{n\to\infty} f_n(x) = f(x)\;(x \in E)$ and $M_n = \sup_{x\in E} \left| f_n(x) - f(x) \right|$ prove that $f_n \to f$ uniformly on E if and only if $M_n \to 0$ as $n \to \infty$. Hence prove that the seq $f_n(x) = \dfrac{\sin nx}{\sqrt{n}}\;(x, real, n = 1,2......)$ converges uniformly on R.

(b) Suppose $\{f_n\}$ is a sequence of functions defined on a compact set K such that (i) $\{f_n\}$ converges pointwise to a limit function f on k (ii) each f_n and f are continuous on k (iii) $f_n(x) \ge f_{n+1}(x)\;\forall\; x \in K n = 1,2.......$ Then prove that $f_n \to f$ uniformly on K.

..2

8 (a) Suppose $f_n \in R(\alpha)$ on [a, b] for n = 1, 2, ….. and suppose that $f_n \to f$ uniformly on [a, b]. Prove that $f \in R(\alpha)$ on [a, b]. Also prove that

$$\int_a^b f \; d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$$

(b) If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E then prove that $\{f_n\}$ has a subsequence $\{ f_{n_k} \}$ such that $\{ f_{n_k}(x)\}$ converges for every $x \in E$.

9 (a) State and prove Lebseque's monotone convergence theorem.

(b) State and prove factors theorem.

10 (a) If $f \in R$ on [a, b] then prove that $f \in L$ on [a, b]. Also prove that

$$\int_a^b f \; dx = R\int_a^b f \; dx$$

(b) State and prove the Riesz – Fischer theorem.

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Code No. 2870 / CDE / N

FACULTY OF SCIENCE

M.Sc. Previous (CDE) Examination, July 2018

Subject: Statistics

Paper – II
Probability Theory

Time: 3 Hours **Max.Marks: 80**

Note : Answer any five questions from Part–A, each question carries 4 marks, Answer all questions in Part–B, each question carries 12 marks.

PART – A (5x4 = 20 Marks)
[Short Answer Type]

Note: Answer any five of the following questions.

1 In tossing an ideal coin twice, let A be the event that the first toss gives a head and let B be the event that the second toss gives a head. Examine whether A and B are independent.
2 Distinguish between a field and a sigma field.
3 State Levy's continuity theorem. What are its uses?
4 Define a characteristic function (c.f). State three properties of a c.f.
5 Distinguish between Weak Law of large numbers (WLLN) and Strong Law of large numbers (SLLN).
6 When do you say that a sequence of random variables (r.v's) obeys Central Limit Theorem (CLT). State Liapunov's form of CLT.
7 Define a Markov Chain (M.C). Explain its classification.
8 Explain mean recurrence time with respect to a M.C.

PART – B (5x12 = 60 Marks)
[Essay Answer Type]

9 a) i) State and prove Bayes' theorem.
ii) There are three stores S_1, S_2 and S_3 in a city selling bags. The respective percentages of defective bags in them are 10%, 15% and 20%. A customer enters a shop at random and selects a bag. If the bag selected is a good one, find the probability that it belongs to S_3.

OR

b) i) If X is a r.v, show that max (0, X) is also a r.v obtain its distribution function (d.f).
ii) State and prove Markov inequality. Obtain special cases if any.

10 a) i) Let X ~ b (n, p). Obtain the characteristic function (c.f) of X.
ii) Verify whether $(1+t^4)^{-1}$ is a c.f.

OR

b) i) Define various modes of convergence of a sequence of random variables.
ii) Establish the connection between convergence in probability and almost sure convergence.

11 a) i) State and prove Bernoulli's form of WLLN.

ii) Examine whether WLLN is applicable for a sequence of iid r.v's with a common pdf $f(x) = 8(x+2)^{-3}$, $x > 0$.

OR

b) i) State and prove Levy's form of CLT.

ii) Examine whether Levy's form of CLT is applicable for a sequence of iid r.v's with a common pdf $f(x) = 2(x+1)^{-3}$, $x > 0$.

12 a) i) State and prove Chapman – Kolmogorov equations.

ii) $\{X_n, n \geq 0\}$ is a M.C. with state space {0, 1} initial distribution $\left(\frac{1}{3}, \frac{2}{3}\right)$ and the transition probability matrix (tpm)

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

Compute (i) $P(x_o = x_1 = x_2 = x_3 = x_4 = 1)$

(ii) $P(x_o = 0, x_1 = 1, x_2 = 1, x_3 = 0)$

OR

b) i) Obtain the stationary distribution of the M.C in 12a(ii).

ii) $\{X_n, n\geq0\}$ is a M.C. with state space {0, 1), initial distribution $\left(\frac{1}{2}, \frac{1}{2}\right)$ and

$$\text{tpm } P = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Compute (i) $P(x_4 = 1 \mid x_o = 0\ x_1 = 1)$

(ii) the marginal distributions of x_1 and x_2.

13 a) i) Define a probability measure. State and prove the addition rule of probability.

ii) Compute $P(Y_n < \log n$ i.0) when Y_n is the maximum in a random sample of size n from a unit exponential distribution.

OR

b) i) State and prove Khintchine's form of WLLN for the iid case.

ii) Examine whether Kolmogorov's SLLN is applicable for a sequence of iid rv's with a common df $F(x) = 1 - \frac{1}{x^2}$, $x \geq 1$.

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Code No. 2853/ CDE

FACULTY OF SCIENCE

M. Sc.(Previous) (CDE) Examination, July 2018

Subject : Mathematics

Paper – III : Topology and Functional Analysis

Time : 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

1 Let X be a topological space and A an arbitrary subset subset of X. Then show that $\bar{A}$={ x | each neighbourhood of x intersects A}.

2 Show that a topological space is compact if and only if every class of closed sets with empty intersection has a finite subclass with empty intersection.

3 Show that a one to one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism.

4 Show that a topological space is a T_1-space if and only if each point is a closed set.

5 Show that on a finite dimensional vector space X, any norm $\|.\|$ is equivalent to any other norm $\|.\|_o$.

6 If a normed space X is finite dimensional then show that every linear operator on X is bounded.

7 Show that the space ℓ^p with $p \neq 2$ is not a Hilbert space.

8 If Y is a closed subspace of a Hilbert space then show that $Y = Y^{\perp\perp}$.

PART – B (5 x 12 = 60 Marks)
(Essay Answer Type)

9 (a) State and prove Tychonoff's theorem.

OR

(b) Show that a metric space is compact if and only if it is complete and totally bounded.

10 (a) State and prove Tietze extension theorem.

OR

(b) Show that a subspace of the real line IR is connected if and only if it is an interval. In particular, prove that IR is connected.

11 (a) Let $\{x_1, x_2, \ldots\ldots x_n\}$ be a linearly independent set of vectors in a normed space X. Then show that there is a C > 0 such that for every choice of scalars $\alpha_1, \alpha_2, \ldots \alpha_n$ we have

$$\| \alpha_1 x_1 + x_2 \alpha_2 + \ldots. + \alpha_n x_n \| \geq C (|\alpha_1| + |\alpha_2| + \ldots + |\alpha_n|).$$

OR

(b) Let Y and Z be subspace of a normed space X and suppose that Y is closed, proper subset of Z. Then prove that for every real number θ in the inter val (0, 1) there is a $z \in Z$ such that

$$\| z \| = 1 \text{ and } \| z - y \| \geq 0 \text{ for all } y \in Y.$$

12 (a) Let X be an inner product space and $M \neq \phi$ a convex subset which is complete in the metric induced by the inner product. Then show that for every given $x \in X$ there exists a unique $y \in M$ such that

$$\delta = \inf_{\tilde{y} \in M} \| x - \tilde{y} \| = \| x - y \|$$

OR

(b) Let H_1 and H_2 be Hilbert spaces and $h : H_1 \times H_2 \rightarrow K$ be a bounded sequilinear form. Then show that h has a representation $h(x, y) = \langle Sx, y \rangle$ where $S : H_1 \rightarrow H_2$ is a bounded linear operator. Show that S is uniquely determined by h and has norm
$\| s \| = \| h \|$.

13 (a) State and prove Lebesgue's covering Lemma.

OR

(b) Let H_1 and H_2 be Hilbert spaces and $S : H_1 \rightarrow H_2$ and $T : H_1 \rightarrow H_2$ be bounded linear operators and α is any scalar. Then prove that

(i) $\langle T^* y, x \rangle = \langle y, Tx \rangle$

(ii) $(S+T)^* = S^* + T^*$

(iii) $(T^*)^* = T$

(iv) $\|T^*T\| = \|TT^*\| = \|T\|^2$

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Code No. 2858 / CDE / O / BL

FACULTY OF SCIENCE

M. Sc.(Previous) (CDE) Examination, July 2018

Subject : Mathematics

Paper – III : Differential Equations and Complex Analysis

Time : 3 Hours **Max. Marks: 100**

Note : Answer any five questions choosing atleast two questions from each part.

PART – A

1 (a) Solve $(y^2 + yz)dx + (xz + z^2)dy + (y^2 – xy)dz = 0$.

(b) Find the curves represented by the solution of $ydx+ (z – y) dy + xdz = 0$ which lie in the plane $2x – y – z = 1$.

2 (a) Show that $\int_{-1}^{1} (P_n(x))^2 \, dx = \frac{2}{2n+1}$.

(b) Show that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$.

3 (a) Show that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$.

(b) State and prove generating function for $\bar{J}_n$ (x).

4 (a) Explain Charpit's method of solving f(x, y, z, p, q)=0 and hence solve $z = px + qy + pq$.

(b) Solve $(D^2 + D'^2)$ z= cosmx cosny

5 (a) Solve $(D^2 + 2DD' + D'^2)$)z = 2cosy – x siny.

(b) Explain Monge's method of integrating Rr+Ss+Tt=V.

PART – B

6 (a) Define analytic function at a point and on a domain G. Show that $f : G \to C$ defined by f(z) = u + iv is analytic if and only if u and v satisfy C – R equations and u, v possess continuous partial derivatives.

(b) State and prove Cauchy's integral formula in general form.

7 (a) Establish Cauchy's estimate and hence deduce Liouville's theorem.

(b) Define Mobius transformation and show that a Mobius transformation takes circles onto circles.

8 (a) State and prove Cauchy Residue theorem.

(b) Prove that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)} = \frac{\text{®}}{3}$.

9 (a) Obtain the Taylor series of $\frac{z^2 - 1}{(z+2)(z+3)}$ in the open disk |z| < 2.

(b) If p(z) is a non constant polynomial then show that there is a complex number a such that p(a) = 0.

10 (a) State and prove maximum modulus theorem.

(b) State and prove Rouche's theorem.

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Code No. 2871 / CDE / N

FACULTY OF SCIENCE

M.Sc. Previous (CDE) Examination, July 2018

Subject: Statistics

Paper – III

Distribution Theory & Multivariate Analysis

Time: 3 Hours **Max.Marks: 80**

PART – A (5x4 = 20 Marks)
[Short Answer Type]

1 Derive the mean and variance of negative binomial distribution.
2 Define K-parameter exponential family of distribution and give example in case of K=2.
3 Define non-central F-distribution and state its properties.
4 Obtain the joint distribution of two order statistics $X_{(r)}$ and $X_{(s)}$.
5 State and prove the additive property of Wishart distribution.
6 Give the null and non-null distribution of multiple correlation coefficients.
7 State the assumptions in the orthogonal factor model.
8 Establish the relationship between T^2 and D^2.

PART – B (5x12 = 60 Marks)
[Essay Answer Type]

9 a) Define Weibull distribution and obtain its characteristic function. Also obtain the mean and variance from it.

OR

b) i) Derive the characteristic function of Pareto distribution.
ii) State the properties of double exponential distribution.

10 a) Define the Truncated Normal distribution truncated at both the ends. Obtain its mean and variance.

OR

b) Derive the pdf of central F-distribution.

11 a) Obtain the maximum likelihood estimates of mean vector and covariance matrix of a multivariate normal distribution.

OR

b) Show that sample mean vector and covariance matrix of a multivariate normal are independent.

12 a) Derive the null distribution of Hotelling T^2 statistics.

OR

b) Derive the canonical variables and canonical correlations.

13 a) Derive the compound Poisson distribution and obtain its mean and variance.

OR

b) Obtain the linear discriminant function using maximizing the likelihood ratio approach.

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Code No. 2854/ CDE

FACULTY OF SCIENCE

M. Sc.(Previous) (CDE) Examination, July 2018

Subject : Mathematics

Paper – IV : Elementary Number Theory

Time : 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

1 Prove that there are infinitely many prime numbers.

2 For $n \geq 1$ prove that $\log n = \sum_{d|n} \wedge(d)$.

3 State and prove Euler-Fermat theorem.

4 Solve the congruence $5x \equiv 3 \pmod{24}$.

5 For every odd prime p prove that $\left(\frac{2}{p}\right) = (-1)^{\frac{p^2-1}{8}}$.

6 Determine those odd primes p for which 3 is a quadratic residue.

7 Define Jacobi symbol $\left(\frac{n}{P}\right)$. Show that $\left(\frac{n}{P}\right)\left(\frac{n}{Q}\right) = \left(\frac{n}{PQ}\right)$.

8 Let p(0) = 1 and define p(n)=0 if n < 0. Then for $n \geq 1$ prove that
p(n) – p(n–1) –p(n–2)+p(n–5)+p(n–7)+………=0

PART – B (5 x 12 = 60 Marks)
(Essay Answer Type)

9 (a) State and prove fundamental theorem of arithmetic.

OR

(b) If both g and $f * g$ are multiplicative then prove that f is also multiplicative.

10 (a) State and prove Chinese remainder theorem and solve the system of congruences

$x \equiv 1 \pmod 3$, $x \equiv 3 \pmod 4$, $x \equiv 2 \pmod 5$.

OR

(b) (i) For any prime p prove that all the coefficients of the polynomial
$F(x) = (x - 1)(x - 2) \ldots (x - p + 1) - x^{p-1} + 1$ are divisible by p.

(ii) State and prove Wilson's theorem.

11 (a) State and prove Gauss Lemma.

OR

(b) Let p be an odd prime. If g is a primitive root mod p then prove that g is also a primitive root mod p^α for all $\alpha \geq 1$ if and only if $g^{p-1} \not\equiv 1 \pmod{p^2}$.

12 (a) State and prove Euler's pentagonal number theorem.

OR

(b) State and prove Jacobi's triple product identity.

13 (a) (i) State and prove Mobius inversion formula.

(ii) If f is multiplicative then prove that $\sum_{d|n} «(d) f(d) = \prod_{p|n} (1 - f(p))$

OR

(b) State and prove quadratic reciprocity law.

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Code No. 2859 / CDE / O / BL

FACULTY OF SCIENCE

M. Sc.(Previous) (CDE) Examination, July 2018

Subject : Mathematics

Paper – IV : Theory of Ordinary Theorem Differential Equations

Time : 3 Hours **Max. Marks: 100**

Note : Answer any five questions. All questions carry equal marks.

1 (a) If ϕ_1 is a solution of $L(Y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I and if $\phi_1(x) \neq 0$ on I then show that its second solution is

$$\phi_2(x) = \phi_1(x)\int_{x_o}^{x}\left\{\frac{1}{[\phi_1(S)]^2}\exp[\int_{x_o}^{x} a_1(t)dt)]\right\} ds$$

(b) Show that there exist n linearly independent solutions of $L(y) = 0$ on I.

2 (a) Let $\phi_1 \ldots\ldots\phi_n$ be n solutions of $L(Y)=0$ on I satisfying $\phi_i^{(j-1)}(x_o) = 1, \; \phi_i^{(j-1)} = 0 \quad j \neq i$ then prove that there are n constants $C_1, \ldots\ldots C_n$ such that $\phi = C_1\phi_1 + C_2\phi_2 \ldots\ldots C_n\phi_n$

(b) If $b_1, \ldots b_n$ are non-negative constants such that $| a_i(x) | \leq b_i$ for all $1 \leq i \leq n$ and $k = 1 + b_1 + \ldots + b_n$ and if x_o is a point in I and ϕ is a solution of $L(Y) = Y^{(n)} + a_1(x) Y^{(n-1)} \ldots + a_n(x)Y = 0$, on I then show that $\| \varphi(x_o) \| e^{-k|x-x_o|} \leq \| \varphi(x) \| \leq \| \varphi(x_o) \| e^{k|x-x_o|}$ for $x \in I$.

3 Let of be a real-valued continuous function on the strip S : $| x – x_o| \leq a$, $| y | < \infty$, $(a > 0)$, and suppose that f satisfies on S a Lipschitz condition with constant $K > 0$. Show that the successive approximations $\{\varphi_k\}$ for the problem $y' = f(x, y)$, $y(x_o) = y_o$ exist on the entire internal $|x - x_o| \leq a$ and converge to a solution φ if above IVP.

4 Let f, g be continuous on R, and suppose f satisfies a Lispchitz condition there with Lipschitz constant K. Let , φ, Ψ be solutions of $y' = f(x, y)$, $y(x_o) = y_1$ and $y' = g(x, y)$, $y(x_o) = y_2$ respectively on an Interval I containing x_o, with graphs contained in R with the equalities $|y(x, y) - g(x, y)| \leq \in$ and $| y_1 – y_2| \leq \delta$ then show that $| \varphi(x) - \Psi(x) | \leq \delta\, e^{k|x-xo|} + \frac{\in}{k}[e^{k|x-x_o|} - 1]$ for all x in I.

5 Let f be a continuous vector-valued function defined on S : $|x - x_o| \leq a$, $|y| < \infty$ $(a > 0)$ and satisfy there a lipschitz condition, then prove that the successive approximations $\{\varphi_k\}$ of the problem $y' = f(x, y)$, $y(x_o) = y_o$ ($| y_o | < \infty$), exist on $|x - x_o| \leq a$, and converge there to a solution φ of this problem.

6 Let $a_1, \ldots a_n$, b be continuous complex-valued functions on an Interval I containing a point x_o. If $\alpha_1, \ldots\alpha_n$ are any 'n' constants then show that there exists one, and only one solution φ of the equation $y^{(n)} + a_1(x)y^{(n-1)} + \ldots + a_n(x)y = b(x)$ on I satisfying $\varphi(x_o) = \alpha_1$, $\phi'(x_o) = \alpha_2$, $\ldots, \varphi^{(n-1)}(x_o) = \alpha_n$.

7 (a) Find adjoint equation of $L(Y) = a_o(x)y' + a_1(x)y = 0$.
(b) Show that every second order linear differential equation $a(x)y''+b(x)y'+(x)y=0$ can be written into self adjoint form.

8 (a) If z, z_1, z_2, z_3 are any four different solutions of Riccatis equation $z^1 + a(x)z + b(x)z^2+C(x)=0$ then show that

$$\frac{(z-z_2)}{(z-z_1)} \bullet \frac{(z_3-z_1)}{(z_3-z_2)} = \text{constant}$$

(b) Define Green's function for the equation $a_o(x)y'' + a_1(x)y' + a_2(x)y = 0$.

9 (a) If u(x) and v(x) $\in C'[a, b]$ and if v(z) = v(b) =0, v(a) $\neq$ 0 on (a, b) also if $w(x) = u(x)\, v^1(x) - v(x)u^1(x)$ is not zero on [a, b] then prove that u(x) vanishes precisely once in (a, b).
(b) Prove that between every pair of consecutive zeros of sinx there is precisely one zero of sinx + cosx.

10 (a) State and prove Bocher-Osgood theorem.
(b) If Y(x), z(x) are solutions of $x^2y''+xy'+(x^2-1)y=0$, $xz''+xz'+xz=0$ which vanish at x=1 then find which solution vanishes first after x=1.

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Code No. 2872 / CDE

FACULTY OF SCIENCE

M.Sc. Previous (CDE) Examination, July 2018

Subject: Statistics

Paper – IV

Sampling Theory & Theory of Estimation

Time: 3 Hours **Max.Marks: 80**

PART – A (5x4 = 20 Marks)
[Short Answer Type]
Note: Answer any five of the following questions.

1 What are the main steps involved in a sample survey? Discuss them briefly.

2 Prove that in SRSWOR, sample mean square is an unbiased estimate of population mean square.

3 Define ratio estimator for estimating the population total of a character Y and derive an expression for the standard error of the estimator.

4 What are the sources of sampling errors? Explain them briefly.

5 If T is an unbiased estimator of θ, then show that T^2 is a biased estimator of θ^2.

6 Describe the properties of a good estimator.

7 State the properties of Maximum Likelihood Estimator (MLE).

8 Let $f(x) = \frac{1}{\theta}$, $0 \le x \le \theta$. Obtain the MLE for θ.

PART – B (5x12 = 60 Marks)
[Essay Answer Type]

9 a) A stratified random sampling with $n_i = \frac{W_i S_i}{\sqrt{\mu_o c_i}}$ where μ_o is constant, c_i is the cost per unit in i^{th} stratum, $W_i = \frac{N_i}{N}$, S_i^2 is the mean square based on N_i units. Estimate the sample size n under optimum allocation for fixed cost C_o.

OR

b) Obtain an unbiased estimator of population variance in case of SRSWOR.

10 a) State and prove Horwitz – Thompson estimator.

OR

b) Distinguish between Ratio Estimation and Regression Estimation.

11 a) State and prove Neymann – Factorization Theorem.

OR

b) What is completeness of distributions? State and prove Lehmann – Scheffe Theorem.

12 a) Show that MLE's are asymptotically normal.

OR

b) Obtain $100(1-\alpha)\%$ large sample confidence interval for the parameter μ of the normal distribution, where σ^2 is known.

13 a) Explain the pivotal method. Construct confidence interval for the parameter of an exponential distribution.

OR

b) Obtain the MLE of θ from $f(x;\theta) = \theta x^{-(\theta+1)}$; $0 < x < 1$, show that the variance of the MLE is lower than Cramer – Rao lower bound and MVUE is non-existent. Explain the reason.

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Code No. 2855/ CDE

FACULTY OF SCIENCE

M. Sc.(Previous) (CDE) Examination, July 2018

Subject : Mathematics

Paper – V : Mathematical Methods

Time : 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

1 Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

2 Prove that $P_{2m}(0) = (-1)^m \frac{(2m)!}{2^{2m}(m!)^2}$.

3 Show that the functions $x_1(t)=t^2$, $x_2(t) = t\,|\,t\,|$ are linearly independent on $-\infty < t < \infty$.

4 Find e^{tA} if $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

5 Show that $f(x, y) = xy^2$ satisfies the Lepchitz condition on the rectangle $|x| \leq 1, |y| \leq 1$ but does not satisfy a Lepchitz condition on the strip $|x| \leq 1, |y| < \infty$.

6 Define Sturm – Liouville problem.

7 Find the complete integral of

$$z^2p^2y + 6zpxy + 2zqx + 4x^2y = 0.$$

8 Solve $r - 2s + t = \sin(2x + 3y)$.

PART – B (5 x 12 = 60 Marks)
(Essay Answer Type)

9 (a) Solve in series the differential equation

$$(1 - x^2)y'' - 2xy' + n(n+1)y = 0.$$

OR

(b) State and prove the orthogonality property of Bessel functions.

10 (a) State and prove Abel's formula.

OR

(b) Let A (t) be a n x n continuous matrix defined on an interval I. If a solution matrix ϕ satisfies $X^1 = A(t)X$, $t \in I$ then show that $(\det \phi)' = (t\,r\,A) \det \phi$.

11 (a) Let f be continuous function defined on the rectangle
R : | $t - t_o$ | ≤ a, | $x - x_o$ | ≤ b, (a, b > 0). Then show a function ϕ is a solution of the IVP $x^1 = f(t, x)$, $x(t_o) = x_o$ on the interval I containing the point t_o if and only if it is solution of the integral equation $x(t) = x_o + \int_{t_o}^{t} f(s, x(s))\, ds$.

OR

(b) Prove that all the eigen values of Sturm – Liouville problem are real.

12 (a) Use the Charpit's method to find the complete integral of

$$16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0.$$

OR

(b) Solve the one dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$ with u(0, t) = u(ℓ, t) = 0.

13 (a) State and prove generating function for Legendre polynomials.

OR

(b) Prove that $\int_{-}^{\infty} e^{-x} L_n(x)\, L_m(x)\, dx = \begin{cases} 0 & if \quad m \neq n \\ 1 & if \quad m = n \end{cases}$

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Code No. 2873 / CDE

FACULTY OF SCIENCE

M.Sc. Previous (CDE) Examination, July 2018

Subject: Statistics

Paper – I (Practical)

Linear Algebra, Distribution Theory & Multivariate Analysis

Time: 3 Hours **Max.Marks: 100**

Note: Answer any three questions. All questions carry equal marks.
Scientific Calculators are permitted.

1 Compute the Moore Penrose inverse of the following matrix:

$$\begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$$

2 Obtain the spectral decomposition of the following matrix:

$$\begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

3 The following data relates to the number of accidents to 750 women working on highly explosive shells during 5-weeks period. Fit an appropriate distribution. Also test for goodness of fit.

No. of accidents	0	1	2	3	4	5
No. of women	475	152	61	40	19	3

4 The following table gives the frequency distribution of heights of 300 students. Fit normal distribution using areas method and test for its goodness of it.

Height (in Inches)	62-63	63-64	64-65	65-66	66-67	67-68	68-69	69-70
Frequency	1	2	1	4	12	31	31	47
Height (in Inches)	70-71	71-72	72-73	73-74	74-75	75-76	76-77	77-78
Frequency	48	42	35	21	14	8	2	1

5 Consider two groups in the city $\prod_1$ riding-mover owners and $\prod_2$: without riding-movers or non-owners. In order to identify the best sales prospects for an intensive sales campaign a riding mover manufacture is interested in classifying families as prospective owner or non-owners on the basis of x_1 = income and x_2 = lot size. Random samples of n_1=6 current owner and n_2=6 current non-owners yield the values in the following data:

$\prod_1$ riding-mover owners		$\prod_2$: without riding-movers or non-owners	
x_1 (income $000)	x_2 (in 1000 sqft)	x_1 (income $000)	x_2 (in 1000 sqft)
36.0	8.8	19.8	8.0
27.6	11.2	22.0	9.2
23.0	10.0	15.8	8.2
31.0	10.4	11.0	9.4
17.0	11.0	17.0	7.0
27.0	10.0	21.0	7.4

Assuming the populations are bivariate normal having common dispersion matrix, test whether $\prod_1$ and $\prod_2$ have the same mean vector or not using Hotelling's T^2 . Test at 5% level.

6 In a diabetic centre the fasting blood sugar levels of two groups of patients are recorded two times, one before the treatment (x_1) and another after the treatment (x_2).

Group-1 : <40 years		Group-2: 40+years	
x_1	x_2	x_1	x_2
172	174	329	310
222	200	314	303
110	206	228	343
233	218	215	311
366	148	153	215
181	366	179	303
185	205	156	130

Carry out the discriminant analysis to find the Fisher's linear discriminant function and allocate the new observation $\underline{X}$ = [180 300]' to an appropriate group.

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Code No. 7951/CDE/N

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July / August 2019

Subject: Mathematics
Paper - I : Algebra

Time: 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5 x 4 = 20 Marks)

1. Define G-set. Show that a group G is a G-set under the action $a * x = a\,x\,a^{-1}$ for all a, $x \in G$.
2. If G is a group of order 108 then show that there exists a normal subgroup of order 27 or 9.
3. State and prove the fundamental theorem of homomorphism of rings.
4. Show that the ring of integers $\mathbb{Z}$ is a Euclidean domain.
5. If M is a simple R-module then prove that Hom_R (M, M) is a division ring.
6. Determine the minimal polynomials of $\sqrt{2}+5$ and $3\sqrt{2}+5$ over Q.
7. If $f(x) \in F[x]$ is an irreducible polynomial over a finite field F then prove that all the roots of $f(x)$ are distinct.
8. If F is a field of characteristic $\neq 2$ and $x^2 - a \in F[x]$ is an irreducible polynomial over F then prove that its Galois group is of order 2.

PART – B (5 x 12 = 60 marks)

9. a) If G is a solvable group then prove that every subgroup of G and every homomorphic image of a are solvable. Conversely if N is a normal subgroup of G such that N and G/N are solvable then G is solvable.

OR

b) State and prove fundamental theorem of finitely generated abelian groups.

10. a) If R is a commutative principal ideal domain with unity then prove that any non zero ideal $P \neq R$ is prime if and only if it is maximal.

OR

b) Let R = F[x] be a polynomial ring over a commutative integral domain F and f(x), g(x) $\neq$ 0 be polynomials in F[x] of degrees m and n respectively. Let $k = \max\{m-n+1,\ 0\}$ and a be the leading co-efficient of g(x) then prove that there exists unique Polynomials q(x) and r(x) in F[x] such that $a^k f(x) = q(x)g(x) + r(x)$ *where* $r(x) = 0$ or $r(x)$ has degree less than the degree of $g(x)$.

11. a) (i) Prove that the submodules of the quotient module M/N are of the form U/N where U is a submodule of M containing N.

(ii) If A and B are R-submodules of R-modules M and N respectively then prove that $\frac{M \times N}{A \times B} \simeq \frac{M}{A} \times \frac{N}{B}$

…..2

OR

b) If E is an algebraic extension of a field F and σ : F→L is an embedding of F into algebraically closed field L then prove that σ can be extended to an embedding ¦ $: E \to L$

12. a) State and prove fundamental theorem of Galois theory.

OR

b) Prove that f(x) ∈ F[x] is solvable by radicals over F if and only if its splitting field E over F has solvable Galois group.

13. a) Prove that any two composition series of a finite group are equivalent.

OR

b) If E is a finite separable extension of a field F then prove that E is a simple extension of F.

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Code No. 7956/CDE/O/BL

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July/August 2019

Subject: Mathematics

Paper - I : Algebra

Time: 3 Hours **Max. Marks: 100**

Note: Answer any five from the following questions.
All questions carry equal marks.

1. a) If ¤ is a homomorphism of G onto $\overline{G}$ with kernel K then prove that $\frac{G}{K} \simeq \overline{G}$

   b) State and Prove Sylow's first theorem.

2. If G is a finite group then prove that $c_a = \frac{O(G)}{O(N(a))}$

3. If O(G) = p^2 where p is a prime prove that G is abelian.

4. Prove that every integral domain can be embedded in a field.

5. (a) If R is a Euclidean ring and a, b $\in$ R where b≠0 is not a unit is R then prove that d(a) < d(b)

   (b) If f(x) and g(x) are primitive polynomials then prove that f(x) g(x) is also a primitive polynomial.

6. (a) If a, b $\in$ K are algebraic over F then prove that a±b, ab and $\frac{a}{b}$ where b ≠0 are all algebraic over F.

   (b) If f[x] $\in$ F[x] then prove that there is a finite extension E of F in which f(x) has a root, and [E:F] ≤ deg. f(x)

7. If F is of characteristic 0 and if a, b are algebraic over F then prove that there exists an element c $\in$ F (a, b) such that F(a,b) = F(c).

8. Let K be the splitting field of f(x) $\in$ F[x] and G(K,F) be its Galois group. Let T be any subfield of K containing F onto the set of sub groups of G(K, F) such that

   (i) $T = K_{G(K,T)}$

   (ii) $H = G(K, K_H)$

   (iii) $[K:T] = 0\,(G(K,T))$ and [T: F] = index of G(K, T) in G(K, F)

   (iv) T is a normal extension of F if and only if G(K, T) is a normal subgroup of G(K, F)

   (v) If T is a normal extension of F then G(T, F) is isomorphic to G(K, F)/ G(K, T)

9. a) State and prove Schrerier's theorem.

   b) If L is a complemented modular Lattice and a, b $\in$L with a ≥ b then prove that there is an element $b_1 \in$L such that $b_1 \leq$ a, b∨b_1 = a and b∧b_1 = 0

10. Prove that Boolean algebra and Boolean ring with identity are equivalent.

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Code No. 7969/CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, July/August 2019

Subject : Statistics

Paper – I : Mathematical Analysis & Linear Algebra

Time : 3 Hours **Max. Marks: 80**

Note: Answer any five from Part – A and answer all the questions from Part – B using internal choice.

PART – A (5x4 = 20 Marks)

1. Examine the function $f(x) = x^2 \cos(1/x)$ if $x \neq 0$
   $= 0$ if $x = 0$
   is of bounded variation on [0,1].
2. Define Riemann-stieltjes integral.
3. State chain rules for Jacobians.
4. State the properties of line integrals.
5. Define Moore-penrose Inverse.
6. State Hermite Form of a matrix and write an example.
7. Find the matrices for the Quadratic form $2x_1^2 + 8x_1x_2 + 6x_2^2$.
8. Define index and signature of a Quadratic form.

PART – B (5x12=60 Marks)

9. (a) i) State and prove the First Mean value Theorem for R-S integrals.
   ii) Investigate the nature of critical points of $f(x,y) = x^4+y^4-x^2-y^2+1$.

OR

   (b) i) Define Jarobian function of n variables.
   ii) If $x^2+y^2+u^2-v^2=0$ and $uv+xy=0$, find the Jarobian of transformation and the derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial g}{\partial y}$.

10. (a) Define Power series, Taylor's series, Laurent's series zeros and poles.

OR

   (b) State and prove the Cauchy-residue Theorem.

11. (a) If A^+ is the Moore penrose in verse of A then show that
   (i) $(A^+)^+=A$ (ii) $P(A) = P(A^+)$

OR

   (b) A solution to the system AX=b exists off the rank of the coefficient matrix A is equal to the rank of the augmented matrix B.

12. (a) Show that the ranges of values of two congruent Quadratic Forms (Q.F's) are same.

OR

   (b) Find the rank, index, canonical form to the Quadratic form
   $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_2x_3 + 2x_1x_3 + 2x_1x_2$.

13. (a) If $A=(a_{ij})$ is positive definite then show that $|A| \leq a_{11}\, a_{22} \,.....a_{mm.}$

OR

   (b) State and prove Cauchy-Schwartz and Hadamard inequalities.

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Code No. 7952/CDE/N

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July / August 2019

Subject: Mathematics
Paper: II Real Analysis

Time: 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

1. Prove that every closed subset of a compact set is compact.
2. Prove that a set E is open if and only if its complement is closed.
3. Prove that a continuous mapping defined on a compact metric space is uniformly continuous.
4. If f is monotonic on (a, b) then prove that the set of points (a,b) at which f is discontinuous is at most countable.
5. With usual notations prove that $\underline{\int_{a}^{b}} f d\sim \le \int_{a}^{\overline{b}} f d\sim$
6. If f is continuous on [a, b] then prove that $f \in R(\sim)$ on [a, b]
7. Give an example of a series of continuous functions whose sum function is discontinuous.
8. State and prove contraction principle.

PART – B (5 x 12 = 60 marks)
(Essay Answer Type)

9. a) Define compact set. Prove every K-call is compact.

OR

b) If E is a subset of R^k then prove that the following are equivalent.
(i) E is closed and bounded.
(ii) E is compact.
(iii) Every infinite subset of E has a limit point in E.

10. a) Suppose (X, d) is a metric space and f, g are real valued functions defined on ECX. Suppose 'P' is a limit point of E. let $\lim_{x \to p} f(x) = q$ and $\lim_{x \to p} g(x) = q'$ then

prove (i) $\lim_{x \to p} (f+g)(x) = q + q'$

(ii) $\lim_{x \to p} (fg)(x) = qq'$

OR

b) If f is monotonically increasing on (a, b) then prove $f(c+)$ and $f(c-)$exists at every point c of (a, b) in fact $\sup_{a<t<c} f(t) = f(c-) \le f(c) \le f(c+) = \inf_{c<t<b} f(t)$ further if a<c<d<b then prove $f(c+) \quad f(c-)$

11. a) suppose F and G are differentiable functions on [a, b], $F' = f \in R$ and $G' = g \in R$.

Then prove $\int_a^b F(x)g(x)dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx$

OR

b) if ¥′ is continuous on [e, b] then prove ¥ is rectifiable and $\Lambda(¥) = \int_a^b |¥'(t)| \; dt$

12. a) Suppose $f_n \to f$ .uniformly on a set E in a metric space let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t) = A_n$ (n = 1, 2, 3,……). Then prove {A_n} converges and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$

OR

b) Show that there exists a real continuous functions on the real line which is no where differentiable.

13. a) State and prove Weierstrass approximation theorem.

OR

b) (i) If $A \in L(\mathbb{R}^n, \mathbb{R}^m)$ then prove $\|A\| < \infty$ and A is a uniformly continuous mapping of $\mathbb{R}^n$ into $\mathbb{R}^m$.

(ii) If $A \in L(\mathbb{R}^n, \mathbb{R}^m)$ and B∈($\mathbb{R}^m$, $\mathbb{R}^k$) then prove $\|BA\| \leq \|B\| \; \|A\|$

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Code No. 7957/CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July 2019

Subject: Mathematics

Paper: II Analysis

Time: 3 Hours **Max. Marks: 100**

Note: Answer any five of the following questions, choosing at least two from each part. All questions carry equal marks.

1. a) Suppose A is the set of all sequences whose elements are the digits 0 and 1. Prove that the set A is uncountable.
   b) Suppose $Y \subset X$. Prove that a subset E of y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

2. a) If $\{K_\alpha\}$ is a collection of compact subsets of a metric space X is non-empty. Prove that , the intersection of every finite subcollection of $\{K_\alpha\}$ is non empty. Prove that $\cap K_\alpha$ is no empty.
   b) Explain the construction of Cantor's set.

3. a) Prove that the subsequential limits of a sequence $\{p_n\}$ in a metric space form a closed subset of X.
   b) Prove that the Cauchy product of an absolutely convergent series and a convergent series is convergent and converges to the product of their sums.

4. a) Suppose X, Y, Z are metric spaces, $E \subset X$, f maps E into Y of maps the range of f, f(E) into Z and h is the mapping of E into Z defined by $h(x) = g(f(x))$ $(x \in E)$. If f is continuous at a point $P \in E$ and g is continuous at the point f(p) them prove that h is continuous of P.
   b) Prove that continuous image of a connected set is connected.

5. (a) Suppose f is continuous on [a, b]. Prove that $f \in R(\alpha)$ on [a, b].
   (b) If $f \in R(\alpha)$ on [a,b] and if a < c < b. Prove that $f \in R(\alpha)$ on [a,c] and on [c,b]. Also Prove that

$$\int_a^b f \, d\sim = \int_a^c f \, d\sim + \int_c^b f \, d\sim$$

6. (a) Suppose $C_n \geq 0$ for n = 1, 2,….. and ΣC_n converges, $\{s_n\}$is a sequence of distinct points in (a, b) and

$$\sim(x) = \sum_{n=1}^{\infty} C_n I(x - s_n)$$

   Suppose f is continuous on [a, b] prove that $\int_a^b f d\sim = \sum_{n=1}^{\infty} C_n f(s_n)$

   (b) State and prove the fundamental theorem of Calculus.

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7. (a) State and prove the Weistrass-M test for uniform convergence of a series of functions.

   (b) Suppose f_n f uniformly on a set E in a metric space. Let x be a limit point of E and suppose that

   $\lim_{t \to n} f(t) = A_n (n = 1,2,3,.....)$ prove that $\{A_n\}$ converges and $\lim_{n \to \infty} A_n = \lim_{t \to x} f(t)$

8. a) Suppose K is a compact metric space and $f_n \in ¸ (k)$ from n = 1, 2, 3, ….. and if $\{f_n\}$ converges uniformly on K then prove that $\{f_n\}$ is equicontinuous on K.

   b) If f is a continuous complex function on [a, b] prove that there exists a sequence of polynomials P_n such that $\lim_{n \to \infty} P_n(x) = f(x)$ uniformly on [a,b]

9. a) suppose be additive regular, non-negative and finite on ξ and ECR^p. Define *(E) the outer measure of E. Also prove that

   i) *(A) = (A) for energy A $\in$ ξ

   ii) If E = $\overset{\infty}{\underset{n=1}{U}} E_n$ then μ^*(E) $\leq \sum_{n=1}^{\infty} «^*(E_n)$

   b) Define a measurable function. Suppose $\{f_n\}$ is a sequence of measurable functions on X such that for each x$\in$ X define

   $g(x) = \sup f_n(x) (n = 1,2,3.....)$ *and* $h(x) = \lim_{n \to \sim} \sup f_n(x)$ prove that g and h are measurable functions on X.

10. a) Suppose f is measurable and non negative on X. For A$\in$ $\mathcal{M}$ define

    ϕ(A) = $\int_A fd«$ Prove that ϕ is countably additive on $\mathcal{M}$.

    b) Let $\{\phi_n\}$ be a complete orthonormal set. If $f \in L^2(«)$ and if $f \sim \sum_{n=1}^{\infty} c_n ¤_n$ then prove that $\int_X |f|^2 du = \sum_{n=1}^{\infty} |c_n|^2$

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Code No. 7970/CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, July / August 2019

Subject : Statistics

Paper – II : Probability Theory

Time : 3 Hours **Max. Marks: 80**

Note: Answer any five questions from Part – A and all questions from Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (5x4 = 20 Marks)
(Short Answer Type)

1. State and prove Bayes theorem. What are its applications?
2. Prove that if $E(x^2) < \infty$ then $v(x)=v[E(x/y)] + e[v(x/y)]$.
3. Define characteristic function and state Levy-cramer continuity theorem.
4. Define : (i) convergence in probability and (ii) convergence in distribution. Disucss the implications and counter implications.
5. State and prove Demoiver – Laplace CLT.
6. State kolmogorov's inequality. Mention its applications.
7. Define first entrance probability and first return probability.
8. Define n^{th} order transition probability and explain gamblers ruin problem.

PART – B (5x12=60 Marks)
(Essay Answer Type)

9. (a) i) State and prove chebychev's inequality and
ii) State and prove Markov inequality.

OR

(b) i) Prove that if x is a r.v then $\frac{1}{x}$ is also a r.v.

ii) If $f(x,y)=\begin{cases} 6xy(2-x-y); 0 < x, y < 1 \\ 0 ; 0.w \end{cases}$.

Find $E(x/y)$ and $V(x/y)$.

10. (a) i) Prove $x_n \xrightarrow{L} x \Rightarrow g(x_n) \xrightarrow{l} g(x)$.

ii) Define almost sure convergence and prove that if $x_n \xrightarrow{a.s} x \Rightarrow x_n \xrightarrow{p} x$.

OR

(b) i) Obtain the characteristic function of a r.v $x \sim N(\mu\ \ \sigma^2)$.

ii) Find the density function of a r.v x whose characteristic function $\phi_x(t)=e^{-|t|}$.

11. (a) State and prove Lindberg – Levy's CLT.

OR

(b) i) State and prove kolmogorov's SLLN for a sequence of independent r.v.'s.

ii) Let $\{x_n\}$ be a sequence of independent r.v,'s with $P(x_n= \pm 2^n)=\frac{1}{2}$. Examine SLLN holds for this sequence.

…..2

12. (a) If $\{x_n; n \geq 0\}$ is a Markov chain defined on the state space {0, 1, 2, 3} with TPM

$$P=\begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/4 & 0 & 3/4 & 0 \\ 0 & 0 & 1/5 & 4/5 \end{bmatrix}$$

(i) Classify the states of the chain.
(ii) Find $P[x_1=1, x_2=2]$ and
(iii) Compute $P_{11}^{(2)}, P_{21}^{(2)}$.

OR

(b) Derive a sufficient condition to examine whether a state of a markov chain is persistant or transient.

13. (a) i) State and prove chebycher's WLLN.

ii) For the sequence of r.v.'s such that $P[x_n = \pm 2^n] = \frac{1}{2}$ n≥1, examine whether WLLN holds.

(b) i) Obtain the MGF of poisson r.v with parameter λ.
ii) Show that MGF does not exists for a Cauchy r.v.

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FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July / August 2019

Subject: Mathematics

Paper- III : Topology & Functional Analysis

Time: 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5 x 4 = 20 Marks)

1. Let X be a non-empty set and consider the class of subsets of X consisting of the empty set ϕ and all sets whose complements are countable. Is this a topology on X?
2. Show that a topological space is compact if and only if every class of closed sets with the finite intersection property has non-empty intersection.
3. Show that in a T_2-space, any point and disjoint compact subspace can be separated by open sets, in the sense that they have disjoint neighbourhoods.
4. Show that the components of a totally disconnected space are its points.
5. Let T be a bounded linear operator, then prove that
   (i) $x_n \to x \ \ implies\ Tx_n \to Tx$.
   (ii) N(T) is closed
6. Show that dual space of $\mathbb{R}^n$ is $\mathbb{R}^n$.
7. Let H be a Hilbert space and $T : H \to H$ be a bounded linear operator, then prove that if T is self-adjoint, $\langle Tx, x\rangle$ is real for all $x \in H$.
8. Let H_1, H_2 be Hilbert spaces and $S : H_1 \to H_2$ and $T : H_1 \to H_2$ be bounded linear operators and α is any scalar. Then prove that
   (i) $\langle T^* y, x \rangle = \langle y, Tx \rangle \quad x \in H_1, y \in H_2$
   (ii) $(S+T)^* = S^* + T^*$
   (iii) $(\sim T)^* = \overline{\sim} T^*$

PART – B (5 x 12 = 60 marks)

9. a) (i) Show that a metric space is compact if and only if it is complete and totally bounded.
   (ii) Show that a closed subspace of a complete metric space in compact if and only if it is totally bounded.

OR

b) State and prove Ascoli's Theorem.

10. a) (i) Show that every compact Hausdorff space is normal.
   (ii) Let X be a T_1 space, and show that X is normal if and only if each neighborhood of a closed set F contains the closure of some neighborhood of F.

OR

b) (i) Let X be a Hausdorff space. Prove that if X has an open base whose sets are also closed, then X is totally bounded.
   (ii) Let X be a compact T_2-space. Then prove that X is totally disconnected if and only if it has an open base whose sets are also closed.

11. a) Prove that is Y is a Banach space then B(X, Y) is a Banach space.

OR

b) Show that dual space of l^p is l^q where $1 < p < \infty$ and q is conjugate of p,

(that is, $\frac{1}{p} + \frac{1}{q} = 1$).

12. a) Show that every bounded linear functional f on a Hilbert space H can be represented in terms of the inner product, namely, $f(x) = \langle x, z \rangle$ where z depends on f, is uniquely determined by f and has norm, $\|z\| = \|f\|$.

OR

b) State and prove Ries z representation theorem for Hilbert spaces.

13. a) State and prove Bessel's inequality.

OR

b) (i) Define direct Sum in vector spaces.

(ii) Let Y be any closed subspace of a Hilbert space H. Then Prove that $= Y \oplus Z$ where $Z = Y^{\perp}$.

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Code No. 7958/CDE/BL

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July/August 2019

Subject: Mathematics

Paper- III : Differential Equations & Complex Analysis

Time: 3 Hours **Max. Marks: 100**

Note: Answer any five of the following questions, choosing at least two from each part. All questions carry equal marks.

1. a) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0 \; if \; m \neq n$

   b) Evaluate $\int x^4 J_1(x)dx$.

2. a) Show that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$.

   b) Solve in series $x^2y'' + xy' + (x^2 - n^2)y = 0$.

3. a) State and prove the necessary and sufficient condition for the equation $Pdx + Qdy + Rdz = 0$ to be exact.

   b) Find the curves represented by the solution of $ydx + (z - y)dy + xdz = 0$.

4. a) Explain Charpit's method and hence solve $(p^2 + q^2)y = qz$.

   b) Find the surface satisfying $r = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$.

5. (a) Solve $(D^2 - 6DD' + 9D'^2)Z = 12x^2 + 36xy$.

   (b) Explain Monge's method of integrating
   Rr+Ss+Tt = v

6. (a) Derive necessary condition for f(z) to be analytic in Polar coordinates.

   (b) Find the radius of convergence of the series.

   (i) $\sum_{n=0}^{\infty} n^n z^n$ (ii) $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ (iii) $\sum_{n=1}^{\infty} \frac{z^{2n}}{2^n}$

7. a) Show that if $f(z) = u + iv$ is analytic then the curves $u(x, y) = c_1$ and $v(x, y) = c_2$ form an orthogonal system of curves.

   b) Show that the mapping $w = \frac{1+z}{1-z}$ maps $|\, z \,| \leq 1$ onto the half plane $\text{Re}(w) \geq 0$

8. a) State and prove Cauchy's integral formula.

   b) Suppose G is an open set and $f : G \to C$ is differentiable then prove that f is analytic in G.

9. a) Discuss various types of singularities.

   b) Using Residues show that $\int_0^{2\pi} \frac{d\theta}{1+a\cos\theta} = \frac{2\pi}{\sqrt{1-a^2}}$

10. a) State and prove argument principle.

    b) State and prove maximum modulus theorem.

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Code No. 7971/CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, July/August 2019

Subject : Statistics

Paper – III : Distribution Theory & Multivariate Analysis

Time : 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5x4 = 20 Marks)
(Short Answer Type)

1. Explain Pareto distribution and find its mean and variance.
2. Define lognormal distribution and state its properties.
3. If X, Y are independent U(0,1), then find p.d.f. of XY.
4. Find the mean and variance of the normal distribution truncated on left side at x=A. Give an illustration.
5. If a multivariate normal vector is partitioned into two sub vectors, which are uncorrelated, then show that the two sub vectors are independent multivariate normal vectors.
6. define Hotelling's T^2 statistic and Mahalnobi's D^2 statistic and establish their relationship.
7. Define principal components and state its importance and applications.
8. Describe factor analysis and give orthogonal factor model.

PART – B (5x12=60 Marks)
(Essay Answer Type)

9. (a) i) Explain gamma distribution and obtain its moment generating function.
   ii) Derive the moment generating function of exponential distribution and hence obtain it's mean and variance.

OR

   (b) i) Derive the characteristic function of Beta distribution of first kind and obtain its characteristic function. Hence find it's mean and variance.
   ii) Define the joint, marginal and conditional probability density functions. Obtain the characteristic function of Laplace distribution and hence find mean and variance.

10. (a) i) Define the compound Poisson distributions and also derive the mean and variance of the distribution.
   ii) Define truncated exponential distribution. Derive its mean and variance.

OR

   (b) i) Define order statistics. Obtain the marginal distribution of r^{th} order statistic.
   ii) Derive the distribution of ¡^{2}. Find its mean and variance.

11. (a) Define multivariate normal distribution. Obtain the maximum likelihood estimate of parameters μ and Σ.

OR

   (b) Prove that the conditional distribution obtained from the multivariate normal distribution is also multivariate normal.

12. (a) Describe the concept of discriminant analysis. Derive linear discriminant function and hence describe the classification between two multivariate populations.

OR

(b) Explain the objective of cluster analysis. Derive the hierarchical clustering methods namely single linkage, complete linkage and average linkage.

13. (a) i) Discuss a test procedure for testing the equality of mean vectors of two multivariate normal populations having equal dispersion matrix.

ii) Explain the method of computing the principal components based on the given sample dispersion matrix S.

OR

(b) If $\overline{X}$ and S^2 are the sample mean and sample variance based on a random sample from a normal population, then derive their sampling distributions and show that they are independent.

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Code No. 7955/CDE/N

FACULTY OF SCIENCE

M.Sc. (previous) CDE Examinations, July / August 2019

Subject: Mathematics

Paper: V Mathematical Methods

Time: 3 Hours **Max. Marks: 80**

Note: Answer any five from the following questions

PART – A (5 x 4 = 20 Marks)

1. Explain Frobenius method of solving .
   $\sim(x)y'' + \;(x)y' + ¥(x)y = 0$
2. Show that $J_{1/2}(x) = \sqrt{\frac{2}{®x}} \sin x$
3. Show that the functions $x_1(t) = t^2, x_2(t) = t\,|\,t\,|$ are linearly independent on $-\infty < t < \infty$
4. Find e e^{tA} when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$
5. Solve the IVP $x' = -x, \quad x(0) = 0, \quad t \geq 0$ using successive approximations method.
6. Show that the function of $f(t, x) = e^t x^{1/2}$ does not satisfy Lipschitz condition on S=$\{(t, x) : |\,t\,| \leq 2, \; |\,x \; \leq 1|\}$
7. Solve $\sqrt{p} + \sqrt{q} = 2x$
8. Explain the method of solving $f(p, \; q, z) = 0$

PART – B (5 x 12 = 60 marks)

9. a) Solve in series $2x^2 y'' + \left(x^2 - x\right) y' + y = 0$

OR

b) State and prove Rodrigue's formula for $P_n(x)$

10. a) Let $¤_1(t)$ a solution of $L(x) = x'' + b_1(t)x' + b_2(t)x = 0$ where b1, b2 are continuous on an interval I, and let $¤_1(t) \neq 0$ on I. Let $t_0 \in I$ then show that the second solution $¤_2(t) = ¤_1 \int_{t_o}^{t} \frac{1}{¤_1{}^2(s)} \exp\left[-\int_{t_o}^{t} b_1(u)du\right] ds$

OR

b) State and prove Abel's Formula.

11. a) State and prove existence and uniqueness theorem.

OR

b) Let f be continuous function defined on the rectangle $R : |t - t_o| \leq a, |x - x_o| \leq b$, a, b >0 then show that a function ¤ is a solution of the IVP $x' = f(t, x), \quad x(t_o) = x_o$ on an interval I containing the point t_o if and only if it is solution of the integral equation $x(t) = x_0 + \int_{t_0}^{t} f(s, x(s))ds$

12. a) Solve: $2zx - px^2 - 2qxy + pq = 0$ using Charpit's method.

OR

b) Solve one dimensional wave equation by separation of variable method.

13. a) solve: $\left(D^2 + DD' - 6D'^2\right)z = yCosx$

OR

b) Solve: (i) $\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$

(ii) Form a partial differential Equation by eliminating arbitrary function f from f(x+y+z , $x^2 + y^2 + z^2$)=0

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FACULTY OF SCIENCE

M.Sc. (Previous-Practical) CDE Examination, July / August 2019

Subject : Statistics

Paper – I: Linear Algebra, Distribution Theory & Multivariate Analysis

Time : 3 Hours **Max. Marks: 100**

Note: Answer any THREE questions. All questions carry equal marks.

1. Find the generalized inverse of the following matrix:

$$\begin{bmatrix} -2 & 6 & 4 \\ 1 & -3 & 2 \\ 1 & 5 & 2 \end{bmatrix}$$

2. Obtain the spectral decomposition of the following matrix:

$$\begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

3. Fit an appropriate distribution and test for its goodness of fit to the following data which gives the number of dodders in a sample of clover seeds.

No.of dodders(x)	0	1	2	3	4	5	6	7	8
Observed frequency(f)	56	156	132	92	37	22	4	0	1

4. Fit a normal distribution to the following data and test for its goodness of fit.

Class	60-65	65-70	70-75	75-80	80-85	85-90	90-95	95-100
Frequency	3	21	150	335	326	135	26	4

5. In a diabetic centre the fasting blood sugar levels of a group of patients are recorded two times, one before the treatment (X1) and another after the treatment (X2).

X_1	X_2	X_1	X_2
172	174	329	310
222	200	314	303
110	206	228	343
233	218	215	311
366	148	153	215
181	366	179	303
185	205	156	130

Carry out the principle component analysis for the above data and obtain first principal component and its variance.

6. The following is sample correlations for five stocks:

$$D = \begin{bmatrix} 1 & & & & \\ .58 & 1 & & & \\ .51 & .60 & 1 & & \\ .39 & .39 & .44 & 1 & \\ .46 & .32 & .43 & .52 & 1 \end{bmatrix}$$

Treating the sample correlation coefficients as similarity measures, cluster analyse the stocks using the single linkage and complete linkage methods.

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Code No. 1045 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Mathematics

Paper – I : Algebra

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 If G is a group and X is a G-set then prove that the action of G on X induces a homomorphism $\phi : G \to S_x$.
2 Define normal series and composition series. Give an example of each.
3 Show that any non-zero homomorphism of a field F into a ring R is one-one.
4 Prove that every Euclidean domain is a PID.
5 If R is a ring with unity then prove that an R-module M is cyclic if and only if M $\simeq \frac{R}{I}$ for some left ideal I of R.
6 Show that $x^3 - 5x + 10$ is irreducible order Q.
7 If the multiplicative group F^* of non-zero elements of a field F is cyclic then prove that F is finite.
8 Prove that the Galois group of $x^4 + 1 \in Q[x]$ is the Klein four group.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 If G is a finite group of order p^n where p is a prime and n > 0 then prove the following:
(i) G has a nontrivial center Z.
(ii) $Z \cap N$ is nontrivial for any nontrivial normal subgroup N of G.

10 State and prove second and third Sylow theorems.

11 If R is a non-zero ring with unity and I is an ideal in R such that $I \neq R$ then prove that there exists a maximal ideal M of R such that $I \subseteq M$.

12 Prove that every PID is a UFD but a UFD need not be a PID.

13 If R is a ring with unity and M is an R-module then prove that the following are equivalent.
(i) M is simple
(ii) M $\neq$ (0) and M is generated by any $x \in M$ where $x \neq 0$.
(iii) M $\simeq \frac{R}{I}$ where I is a maximal left ideal of R.

14 If E is an algebraic extension of a field F and $\sigma : F \rightarrow L$ is an embedding of F into an algebraically closed field L then prove that σ can be extended to an embedding η: E→ L.

15 State and prove the fundamental theorem of algebra.

16 State E be a finite extension of F and $f: G \rightarrow E^*$ where $E^* = E - \{0\}$ has the property that $f(\sigma\eta) = \sigma(f(\eta))\, f(\sigma)$ for all $\sigma, \eta \in G$. Then prove that there exists $\alpha \in E^*$ such that $f(\sigma) = \sigma(\alpha^{-1})\alpha$ for all $\sigma \in G$.

17 State and prove Burnside theorem.

18 In a commutative ring R prove that an ideal P is prime if and only if $ab \in P$, $a, b \in R$ implies $a \in P$ or $b \in P$.

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Code No. 1063 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Statistics

Paper – I: Mathematical Analysis & Linear Algebra

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Define function of bounded variation.
2 Show that the set of points of discontinuity of a monotonic function f(x) defined on [a,b] is atmost countable.
3 Explain differentiability at a point.
4 Evaluate $f(x, y) = \int_o^1 \int_o^1 \frac{x - y}{x + y} dx \; dy$.
5 For any matrix A, show that $(A'A)^+ = A^+(A')^+$.
6 Define Moore-Penrose inverse.
7 Show that the matrices A, $P^{-1}AP$ have the same characteristic roots when P is a non-singular matrix.
8 Define algebraic and geometric multiplicity of characteristic roots.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 Let α be a function of bounded variation on [a,b] and assume that $f \in R(a)$. on [a,b]. Then show that $f \in R(a)$ on every subinterval [c,d] of [a,b].

10 If $f \in R(a)$, then show that $\alpha \in R(\alpha)$ on [a,b] and

$$\int_a^b f(x) d\alpha(x) + \int_a^b \alpha(x) df(x) = f(b)\alpha(b) - f(a)\alpha(a)$$

11 State and prove mean value theorem for two variable functions.

12 State and prove Tailors theorem for two variables.

13 Define orthogonal and unitary matrix. Show that column vectors / row vectors of a unitary matrix are normal and orthogonal in pairs.

14 Describe Gram-Schmidt orthogonalization process.

15 Derive spectral decomposition of a real symmetric matrix.

16 Define quadratic forms. State and prove properties of congruent matrices and congruent quadratic forms.

17 Define rank, index and signature of a quadratic forms. Explain classification of quadratic forms.

18 State and prove:
a) Cauchy-Schwartz inequality and
b) Hadamard's inequality.

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Code No. 1048 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub : Mathematics

Paper – IV: Elementary Number Theory

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Show that there are infinitely many primes.

2 Prove that $\phi^{-1}(n) = \sum_{d/n} d\,\mu(d)$.

3 If $c > 0$ then prove that $a \equiv b$ (mod m) if and only if $ac \equiv bc$ (mod mc).

4 If P is an odd prime then prove that $1^{P-1} + 2^{P-1} + \ldots + (P-1)^{P-1} \equiv (-1) \pmod{P}$.

5 Find the quadratic residues modulo 17.

6 Prove that Legendre symbol is completely multiplicative.

7 Prove that $\left(\frac{a^2 n}{P}\right) = \left(\frac{n}{P}\right)$ whenever (a, p) = 1 and P is an odd integer.

8 Evaluate $\left(\frac{219}{383}\right)$.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 (i) Prove that $d(n)$ is odd if and only if n is a square.

(ii) Prove that $\prod_{t/n} t = n^{\frac{d(n)}{2}}$.

10 State and prove generalized Mobius inversion formula.

11 (i) For $n \geq 1$, show that $\sum_{d/n} \phi(d) = n$.

(ii) For $n \geq 1$, show that $\phi(n) = \sum_{d/n} \mu(d) \frac{n}{d}$.

12 If both g and $f * g$ are multiplicative then prove that f is multiplicative.

13 State and prove Lagrange's theorem for polynomial congruences.

14 Prove that the quadratic congruence $x^2+1 \equiv 0$ (mod P) where P is an odd prime has a solution if and only if P≡1 (mod 4).

15 State and prove Gauss Lemma.

16 Let P be an odd prime and d > 0 be such that d divides (P-1). Then prove that in every reduced residue system modulo P there are exactly $\varphi(d)$ numbers "a" such that $\exp_P(a) = d$.
Also prove that there are exactly $\phi(P-1)$ primitive roots modulo P.

17 State and prove Euler's recursion formula for p(n).

18 For complex z and x with |x| < 1 and z≠0, prove that

$$\prod_{n=1}^{\infty}\left(1-x^{2n}\right)\left(1+x^{2n-1}z^{2}\right)\left(1+x^{2n-1}z^{-2}\right)=\sum_{m=-\infty}^{\infty}x^{m^2}z^{2m}.$$

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Code No. 1054 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Mathematics

Paper – IV : Complex Analysis

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Define analytic function. Verify Cauchy-Reimann equations for the functions of z^2 and z^3.

2 Prove that a linear transformation carries circles into circles.

3 Compute $\int_{|z|=2} \frac{dz}{z^2+1}$.

4 Prove that $z = 0$ is not a removable singular point of $f(z) = \frac{\sin z}{z^2}$.

5 If $f(z) = \frac{(3z+1)^4}{(z-1)^2 (z-3)^4}$, then compute the value of $\int_{|z|=4} \frac{f'(z)}{f(z)} dz$.

6 Evaluate the poles and residue at those poles of $f(z) = \frac{z^2}{(z-1)^2 (z-2)}$.

7 Show that $\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right) = \frac{1}{2}$.

8 Show that $\Gamma\left(\frac{1}{6}\right) = 2^{-1/2} \left(\frac{3}{\pi}\right)^{1/2} \left(\frac{\pi}{3}\right)^2$.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 State and prove the sufficient condition for analytic function.

10 Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

11 State and prove Cauchy's integral formula.

12 Evaluate $\int_{|z|=e} \frac{|dz|}{|z-a|^2}$.

13 State and prove Cauchy's residue theorem.

14 State and prove the mean value property for harmonic functions.

15 State and prove Weierstrass theorem.

16 State and prove Legendre's duplication formula.

17 Compute $\int_0^{2\pi} \frac{d\theta}{3 + 2\cos\theta}$ using residues.

18 State and prove Jensen's formula.

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Code No. 1066 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub : Statistics

Paper – IV : Sampling Theory & Theory of Estimation

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Explain the method of drawing a random sample using systematic sampling procedure.

2 What are sampling and non-sampling errors?

3 Explain the cumulative total method of drawing PPS sample with replacement.

4 Compare PPSWOR with SRSWOR.

5 If $x_1, x_2 \ldots x_n$ is a random sample from a $N(\mu,1)$ population then show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$.

6 Explain the concepts of:
i) LMUVE and
ii) UMVUE
with suitable examples.

7 Let X ~ N $(0,\theta)$, show that T(x) = x^2 is complete.

8 Let $x_1, x_2 \ldots x_n$ be a random sample of size n drawn from exponential population with density f(x,θ) = $\theta e^{-\theta x}$, $x \geq 0,\ \theta > 0$ then find the MLE of θ .

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 If the population consists of a linear trend, then show that $V(\overline{Y}_{St}) \leq V(\overline{Y}_{Sys}) \leq V(\overline{Y}_n)_{SRS}$.

10 Prove that in SRSWOR, sample mean is an unbiased estimator of population mean and also obtain the variance of sample mean.

11 Derive Horwitz-Thompson estimator for the population total and find its variance. Also find Yates and Grundy variance estimator.

12 Derive the variance of Regression estimator of population mean in SRS with
i) Pre assigned value of regression coefficient and
ii) Estimated value of regression coefficient.

13 State and prove Cramer-Rao inequality and explain its role in the theory of estimation.

14 Define unbiased estimator. If $x_1, x_2, \ldots, x_n$ ~ N(u, σ^2) then prove that sample mean is an unbiased estimator of μ . Also obtain the unbiased estimator for the population variance.

15 Explain the method of moments for estimating parameters of normal distribution. Comment on the efficiency of these estimators with respect to maximum likelihood estimators.

16 Define:
i) Interval estimation and
ii) Confidence level.
Explain the Pivot method of obtaining a confidence interval. Derive the confidence interval for the parameter μ when the sample is drawn from $N(\mu, \sigma^2)$.

17 In cluster sampling, find an unbiased estimator for the population mean and derive its variance. Also find the relative efficiency of this estimator over SRSWOR.

18 Define complete sufficient statistic and explain its importance in estimation with suitable examples. State and prove Lehman – Scheffe theorem.

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Code No. 1046 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Mathematics

Paper – II: Real Analysis

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Define a countable set. Prove that the set Z of all integers is a countable set.

2 Prove that every compact subset of a metric space is closed.

3 Define Cauchy's product of two series. Give an example to show that Cauchy's product of two convergent series need not be convergent.

4 Suppose f is a continuous function defined on a compact metric space X into a metric space Y. Prove that f(X) is a compact subset of Y.

5 If P^* is a refinement of P with usual notations, prove that $L(P, f, \alpha) \le L(P^*, f, \alpha)$.

6 If $f \in R(\alpha)$ on $[a, b]$ and c is a constant, prove that $cf \in R(\alpha)$ on $[a, b]$ and $\int_a^b (cf)\, d\alpha = c\int_a^b f\, d\alpha$.

7 State and prove Cauchy's criteria for uniform convergence of a sequence of functions.

8 Suppose X is a finite dimensional vector space and A is a linear operator on X. Prove that A is one-one if and only if range of A=X.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 i) Suppose $\{K_\alpha\}$ is a collection of compact subsets of a metric space such that intersection of any finite sub collection is non-empty. Prove that $\bigcap_\alpha K_\alpha$ is non-empty.

ii) Prove that every k-cell is compact in R^k.

10 Prove that a subset E of R is connected if and only if E is an interval.

11 State and prove Riemann's rearrangement theorem.

12 Suppose (X, d) and (Y, ρ) are metric spaces and f: X→ Y is a mapping. Prove that f is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

13 Prove that $f \in R(\alpha)$ on $[a, b]$ if and only if given any ϵ>0 there exists a partition P of $[a, b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

14 Suppose γ is a curve on [a, b] such that γ' is continuous on [a, b]. Prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\,dt$.

15 Suppose $f_n \to f$ uniformly on a set E in a metric space X. Let x be a limit point of E. Suppose $\lim_{t \to x} f_n(t) = A_n$ for every n . Then prove that the sequence $\{A_n\}$ converges. Also prove that $\lim_{n \to \infty} A_n = \lim_{t \to x} f(t)$.

16 State and prove contraction principle.

17 Prove that Cantor's set is (i) compact and (ii) perfect.

18 Prove that on a non-compact bounded subset of R there exists a continuous function which is not uniformly continuous.

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Code No. 1064 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Statistics

Paper – II: Probability Theory

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Define De-Morgan's rules for compliments.
2 State Axiomatic definition of probability.
3 State uniqueness theorem of characteristic function with example.
4 Define convergence in probability and convergence in law.
5 State Kolomogorou's strong law of large numbers for i.i.d random variables and discuss its applications.
6 Define SLLNs, WLLNs and CLT.
7 Define Markov chain, Time homogenous Markov chain and one-step transition probability matrix.
8 Define recurrent and transient states.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 i) Prove that the distribution function of a random variable x is non-decreasing, continuous on the right with $F_x(-\infty) = 0$ and $F_x(+\infty) = 1$. Conversely, every function F with the above properties is the distribution function of a random variable on some probability space.

ii) Let the three dimensional vector $\underset{\sim}{X}$ = (X_1 X_2 X_3) has p.d.f.

$$f_x(x_1\ x_2\ x_3) = \begin{cases} 6x_1\ x_2\ x_3 & if \quad 0 \le x_1 \le 1,\ 0 \le x_2 \le 1,\ 0 \le x_3 \le 2 \\ 0 & \text{Otherwise} \end{cases}$$

Find the marginal p.d.t. of x_1, x_2, x_3 and (x_1, x_3)/

10 i) State and prove Minikowski inequality.
ii) Define expectation and show that, if x and y are one-dimensional random variables then E(x±y) = E(x) ±E(y).

11 i) Define characteristic function. Show that the characteristic function of a normal distributive in $e^{itu} - \frac{t^2\sigma^2}{2}$.

ii) Show that for a characteristic function $|\varphi(t)| \le \varphi(0) = F(+\infty) - F(-\infty)$ and $\varphi(t) = \overline{\varphi}(t)$, where $\overline{\varphi}(t)$ is complex conjugate of $\varphi(t)$.

12 i) State and prove Borel strong law of large numbers.
ii) State and prove Khintchines weak law of large numbers.

13 State and prove Liapunov forms of central limit theorem.

14 State Lindeberg – Feller form of central limit theorem and discuss its applications.

15 i) Discuss classification of Stochastic process with examples.
ii) State and prove Chapman – Kolmogon equation.

16 $\{x_n, n \geq 0\}$ be a M.C. defined on the state space (0, 1, 2) with initial distribution $p\{x_o = i\} = 1/3$, i = 0, 1, 2 and with t.p.m.

$$p = \begin{matrix} 0 \\ 1 \\ 2 \end{matrix}\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$

Find:

i) $p\{x_2 = 2, x_1 = 1, x_o = 2\}$

ii) $p(x_2 = 1 / x_o = 2)$

iii) $p(x_3 = 2 / x_o = 1)$.

17 i) Show that if state j is persistent or recurrent then $\sum_{n=0}^{\infty} p_{jj}^{(n)} = \infty$ or $< \infty$.

ii) Show that if state k is either transient or null persistent then, for every j $p_{jk}^{(n)} \to 0$ as $n \to \infty$ and state k is a periodic, persistent non-null then $p_{jk}^{(n)} \to \frac{F_{jk}}{\mu_{k\mu}}$ as $n \to \infty$.

18 Show that if state j is persistent non-null then

i) $p_{jj}^{(nt)} \to \frac{t}{\mu_{jj}}$ as $n \to \infty$ and when state j is periodic with period t.

ii) $p_{jj}^{(n)} \to \frac{1}{\mu_{jj}}$ when state j is a periodic..

iii) $p_{jj}^{(n)} \to 0$ when state j is persistent null.

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Code No. 1047 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub : Mathematics

Paper – III : Topology & Functional Analysis

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Let T_1 and T_2 be two topologies on a non-empty set X, then prove that $T_1 \cap T_2$ is also a topology on X.

2 Show that continuous image of compact space is compact.

3 Show that compact subspace of Hausdorff space is closed.

4 Show that components of a totally disconnected space are its points.

5 If X is a vector space define

i) Algebraic dual space X^*

ii) Second algebraic dual space X^{**}

iii) Canonical mapping of X into X^{**}.

6 Show that dual space X' of a normed space X is a Banach space.

7 Let H be a Hilbert space and U: H $\rightarrow$ H and V: H $\rightarrow$ H be unitary. Then prove that

i) U is isometric

ii) $||U|| = 1$

iii) UV is unitary.

8 Let H_1 and H_2 be Hilbert spaces and S: $H_1 \rightarrow H_2$ and T: $H_1 \rightarrow H_2$ be bounded linear operators. Then prove that:

(i) $(T^*)^* = T$

(ii) $\|T^*T\| = \|TT^*\| = \|T\|^2$

(iii) $(ST)^* = T^*S^*$

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 a) Show that a metric space is sequentially compact if and only if it has the Bolzano-Weierstrass property.

b) Show that every compact metric space has the Bolzano-Weierstrass property.

10 a) State and prove Tychonoff' theorem.

b) Show that every closed and bounded subspace of $\mathbb{R}^n$ is compact.

11 Let X be a normal space and let A and B be disjoint closed subspaces of X. If $[a, b]$ is any closed interval on the real line then prove that there exists a continuous real function f defined on X, all of whose values lie in $[a, b]$ such that $f(A) = a$ and $f(B) = b$.

12 a) Let X be a topological space. If $\{A_i\}$ is a non—empty class of connected subspaces of X such that $\bigcap_i A_i \neq \phi$, then prove that $A = \bigcup_i A_i$ is also a connected space of X.

b) Let X be a compact T_2-space. Then prove that X is totally disconnected if and only if it has an open base whose sets are also closed.

13 State and prove Baire's category theorem.

14 State and prove uniform boundedness theorem.

15 Let H_1 and H_2 be Hilbert spaces and $h: H_1 \times H_2 \to K$ be a bounded sesquilinear form. Then prove that h has a representation $h(x, y) = \langle Sx, y \rangle$ where $S: H_1 \to H_2$ is a bounded linear operator. Also prove that S is uniquely determined by h and has norm $|| S || = || h ||$.

16 Let $T: H_1 \to H_2$ be bounded linear operator where H_1 and H_2 are Hilbert spaces. Then prove that T^* , the Hilbert adjoint operator of T is unique and is bounded linear operator with $||T^*|| = ||T||$.

17 Let X be an inner product space and $M \neq \phi$, a convex subset which is complete (in the metric induced by the inner product). Then prove that for every given $x \in X$ there exists a unique $y \in M$ such that $\delta = \inf_{y \in M} \|x - \bar{y}\| = \|x - y\|$.

18 (a) If Y is a closed subspace of a Hilbert space H then prove that $Y = Y^{\perp\perp}$.
(b) For any subset $M \neq \phi$ of a Hilbert space H, prove that span of M is dense in H if and only if $M^{\perp} = \{0\}$.

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Code No. 1049 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub : Mathematics

Paper – III : Mathematical Methods

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 If n is positive integer, show that $J_{-n}(x) = (-1)^n J_n(x)$.

2 Find the solution of $y'' - y = x$ using power series method.

3 Find the fundamental matrix for the system $x' = Ax$, where A = $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$.

4 Define Wronskian of n-functions $\phi_1, \phi_2, \dots \phi_n$. Show that the functions $x_1(t) = \sin t$, $x_2(t) = \cos t$ are linearly independent on $-\infty < t < \infty$.

5 Define Green's function.

6 Solve the IVP $x' = 3t^2 x$, $x(0) = 1$ using successive approximations method.

7 Solve: $(D^2 + 2DD' + D'^2)z = e^{3x-4y}$.

8 Solve: $p^2+q^2 = x+y$.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 State and prove orthogonality property of Legendre polynomials.

10 Show that $e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)\, t^n}{n!}$.

11 Let $b_1, b_2, \dots, b_n$ be defined and continuous on an interval I. Then show that any set of n solutions $\phi_1, \phi_2, \dots \phi_n$ of $L(x) = x^{(n)} + b_1(t)x^{(n-1)} + \dots + b_n(t)\, x = 0$ on I are linearly independent on I if and only if $w(t) = w(\phi_1, \phi_2, \dots \phi_n)(t) \neq 0$ for all t in I.

12 Let $b_1, b_2, \dots, b_n$ be continuous on an interval I. Let $\phi_1, \phi_2, \dots \phi_n$ be a basis for the solutions of $L(x) = x^{(n)} + b_1(t)\, x^{(n-1)} + \dots + b_n(t)x = 0\ M$. Then show that a particular solution ψ_p (t) of that equation $L(x) = x^{(n)} + b_1(t)x^{(n-1)} + \dots + b_n(t)x = h(t)$ is given by

$$\psi_p(t) = \sum_{k=1}^{n} \varphi_k(t) \int_{t_0}^{t} \frac{w_2(s)h(s)}{w(s)} ds.$$

13 State and prove contraction principle.

14 Let f be continuous function defined on the rectangle $R: |t - t_0| \leq a, |x - x_0| \leq b, (a, b > 0)$. Then show that a function ϕ is solution of the IVP $x' = f(t,x), x(t_0) = x_0$ on an interval I containing the point t_0 if and only if it is solution of the integral equation $x(t) = x_0 + \int_{t_0}^{t} f(s, x(s))ds$.

15 Find the complete integral of $z^2(p^2z^2+q^2)=1$ also find the singular integral if exists.

16 Solve (i) $(x^2-yz)p+(y^2-zx)q = z^2 - xy$ and (ii) $p^2+q^2=1$.

17 Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$.

18 Solve r + s – 6t = y cosx.

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Code No. 1065 / CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, February 2021

Sub: Statistics

Paper – III: Distribution Theory & Multivariate Analysis

Time: 2 Hours **Max.Marks:80**

PART – A

Answer any four questions. **(4x5=20 Marks)**

1 Derive moment generating function of two parameter gamma distribution.
2 Explain Weibul distribution ad find its mean.
3 Define compound binomial distribution and derive it's mean.
4 Define truncated Poisson and normal distributions. Give an illustration in each case.
5 Define Wishart distribution and establish its additive property.
6 Establish the relationship between multiple and partial correlation coefficients and explain their significance.
7 Define the first two principle components and show that the sum of the variances of all principal components is equal to the sum of the variances of all original variables.
8 Briefly explain the single linkage method.

PART – B

Answer any four questions. **(4x15=60 Marks)**

9 i) Derive moment generating function of normal distribution and hence find the mean and variance.

ii) If $X \sim N(\mu, \sigma)$, then show that $Y = \frac{(X-\mu)^2}{2\lambda\sigma^2}$ is gamma $(\lambda, 1/2)$.

10 i) Derive the characteristic function of Beta distribution of second kind and hence or otherwise find its mean and variance.

ii) Define Cauchy distribution. If X follows Cauchy distribution, then find the p.d.f. for x^2 and identify its distribution.

11 If $\overline{X}$ and S^2 are the sample mean and sample variance based on a random sample from a normal population, then derive their sampling distributions and show that they are independent.

12 i) X, Y are independent uniform variables over (0,1). Find the p.d.f of 1) U = XY and 2) V = X/Y.

ii) If X,Y are iid exponential random variables with parameter θ, then find the distribution of V=X / (X+Y).

13 Show that the sample mean vector and dispersion matrix of the of the multi variate normal population are independently distributed and derive their sampling distributions.

14 Define multivariate normal distribution. Prove that the conditional distribution obtained from the multivariate normal distribution is also multivariate normal.

15 Derive linear discriminant function and hence describe the classification between two multivariate populations.

16 Write in detail about multi-dimensional scaling.

17 Obtain the ML estimators of the parameters of a multivariate normal distribution.

18 i) Define k-parameter exponential family distribution and express the distributions normal and beta 2^{nd} kind in the form of exponential family if exists.

ii) Define t and F distributions and state their properties and give their applications.

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